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Adaptive Parameter Control Using Search State Estimation and Extreme Individuals for Differential Evolution

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Introduction Optimization problems are very important and frequently appear in the real world. There exist many studies on solving optimization problems using population-based optimization algorithms (POAs) such as evolutionary algorithms including differential evolution (DE). The performance of DE is affected by algorithm parameters, mutation strategies and so on. One of the most successful studies on adaptive control of the parameters in DE is JADE (adaptive DE with optional external archive)[1]. In this study, in order to improve the search efficiency of JADE, we propose to improve parameter control in JADE by integrating two methods we have proposed. One is the search state estimation method proposed for particle swarm optimization[2] in which the normalized distance between the center and the best particle is used to estimate the search state. The estimation method need to be adjusted for JADE. The other is the extreme individuals method proposed for JADE[3]. In the extreme individuals method, the parameter values generated by JADE are modified to accelerate the convergence of good individuals and realize global search by bad individuals. The method is simplified to reduce the interference of the two methods. The parameters are modified only for the best individual and the worst individual .

JADE Main algorithm parameters in DE are the scaling factor F and the crossover rate CR. The same values for F and CR are used for all individuals in standard DE. In JADE, the parameter values are generated adaptively for each i-th individual as F_i and CR_i.

Let the population be denoted by $P{=}\{\boldsymbol{x}_1\boldsymbol{x}_2,\cdots,\boldsymbol{x}_N\}$, where $\boldsymbol{x}_i$ is the i-th individual and N is the population size. For each parent $\boldsymbol{x}_i$, a mutation vector $\boldsymbol{m}_i$ is generated by current-to-pbest mutation strategy and a child is generated by recombining $\boldsymbol{x}_i$ and $\boldsymbol{m}_i$. If the child is better than the parent, the child survives in the next generation.

In JADE, the mean value of the scaling factor μ_F and the mean value of the crossover rate μ_{CR} are learned. The scaling factor F_i and the crossover rate CR_i for each individual $\boldsymbol{x}_i$ are independently generated according to the two probability distributions as follows:

$$F_i \sim C(\mu_F,\sigma_F),\ CR_i \sim N(\mu_{CR},\sigma_{CR}^2) \tag{1}$$

where F_i is a random variable according to a Cauchy distribution. CR_i is a random variable according to a normal distribution $N(\mu_{CR},\sigma_{CR}^2)$. CR_i is truncated to $[0,1]$ and F_i is truncated to be 1 if $F_i > 1$ or regenerated if $F_i \le 0$. The location μ_F and the mean μ_{CR} are updated as follows:

$$\mu_F = (1-c)\mu_F + cS_{F^2}/S_F,\ \mu_{CR} = (1-c)\mu_{CR} + cS_{CR}/S_N \tag{2}$$

where S_N is the number of success cases, S_F, S_{F^2} and S_{CR} are the sum of F, F^2 and CR in success cases when a child survives, respectively. The recommended values are $\sigma_F = \sigma_{CR} = 0.1$ and $c = 0.1$.

The search state estimation method In [2], we proposed a method to estimate search state of a population: converging or moving. The center of the population $\boldsymbol{c}$ can be defined as $\boldsymbol{c} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_i$ Let the Euclid distance between two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ be denoted by $d(\boldsymbol{x},\boldsymbol{y})$. The distance between the center $\boldsymbol{c}$ and each individual $\boldsymbol{x}_i$ is given by $d_i = d(\boldsymbol{c},\boldsymbol{x}_i)$. The normalized distance between the center and the best individual, DCB is defined as follows:

$$DCB = \frac{d_{best}-d_{\min}}{d_{\max}-d_{\min}},\ d_{\min} = \arg\min_i d_i,\ d_{\max} = \arg\max_i d_i \tag{3}$$

where d_{best} is the distance between the center and the best individual. To avoid abrupt changes in DCB, the exponentially smoothed moving average with the smoothing constant 0.5 is used.

If the best individual is close to the center, or DCB is small, the population is considered to be converging. If it is far from the center, or DCB is large, the population is considered to be moving. The relationship among DCB, states and parameter control (a simplified version of [2]) are as follows:

- $DCB \in [0,0.05)$: The state is judged as "strongly Converging", and the value of F_i is multiplied by 0.98 to speed up the convergence.
- $DCB \in [0.6,1.0]$: The state is judged as "Moving", and the value of F_i is multiplied by 1.02 to keep the population diversity and accelerate the moving speed.

The extreme individuals method In [3], we proposed parameter control in JADE for extreme individuals where exploitation by the top individuals and exploration by the bottom individuals are realized. In this study, we propose the simplified version of the control:

- Exploitation by the best individual: Local search is realized by adopting small F and large CR: $F_i \sim u(0, \mu_F)$, $CR_i \sim u(\mu_{CR}, 1)$ where $u(l, u)$ is an uniform distribution in $[l, u]$.
- Exploration by the worst individual: Global search is realized by adopting large F and randomized CR: $F_i \sim u(\mu_F, 1)$, $CR_i \sim u(0, 1)$

Experimental results Independent 50 runs are performed for 13 benchmark problems[1] including unimodal and multimodal problems. Table 1 shows the mean value and standard deviation of best objective values over 50 runs for JADE, JADE with search state estimation method, JADE with extreme individuals method, JADE with both method (proposed method). The proposed method attained the best mean result in 9 functions, JADE with extreme individuals attained the best mean result in 2 functions.

Table 1: Experimental result on JADE and the proposed methods

func.	$FE_{\max}$	JADE	JADE+search state	JADE+extreme	proposed
f_1	150,000	9.38e-59 ± 6.5e-58	2.84e-58 ± 2.0e-57	1.04e-66 ± 3.0e-66	**1.16e-69 ± 4.3e-69**
f_2	200,000	4.19e-31 ± 2.4e-30	1.03e-34 ± 7.2e-34	3.22e-43 ± 8.8e-43	**3.05e-46 ± 5.3e-46**
f_3	500,000	8.17e-62 ± 3.0e-61	9.79e-60 ± 5.4e-59	8.93e-62 ± 3.5e-61	**4.72e-62 ± 1.9e-61**
f_4	500,000	2.01e-23 ± 9.8e-23	1.39e-24 ± 4.9e-24	1.03e-26 ± 2.8e-26	**3.67e-27 ± 8.1e-27**
f_5	300,000	5.83e-01 ± 3.6e+00	3.19e-01 ± 1.1e+00	**1.59e-01 ± 7.8e-01**	3.19e-01 ± 1.1e+00
f_6	10000	3.02e+00 ± 1.3e+00	2.00e+00 ± 1.1e+00	2.96e+00 ± 1.6e+00	**1.98e+00 ± 1.1e+00**
f_7	300,000	6.04e-04 ± 2.4e-04	**5.80e-04 ± 2.2e-04**	6.80e-04 ± 2.4e-04	6.44e-04 ± 2.2e-04
f_8	100,000	**2.37e+00 ± 1.7e+01**	4.74e+00 ± 2.3e+01	7.11e+00 ± 2.8e+01	4.74e+00 ± 2.3e+01
f_9	100,000	1.01e-04 ± 3.9e-05	1.28e-04 ± 1.4e-04	4.10e-05 ± 2.8e-05	**3.50e-05 ± 2.9e-05**
f_{10}	50,000	9.20e-10 ± 6.4e-10	3.20e-10 ± 2.1e-10	3.69e-10 ± 2.5e-10	**1.29e-10 ± 7.9e-11**
f_{11}	5,0000	3.17e-07 ± 1.6e-06	1.98e-04 ± 1.4e-03	**1.20e-08 ± 8.4e-08**	2.02e-08 ± 1.4e-07
f_{12}	40,000	2.40e-16 ± 1.6e-15	1.05e-18 ± 2.0e-18	1.75e-18 ± 5.9e-18	**1.13e-19 ± 1.7e-19**
f_{13}	50,000	1.15e-16 ± 2.2e-16	1.34e-17 ± 2.4e-17	9.39e-18 ± 1.4e-17	**8.04e-19 ± 1.8e-18**

Conclusion We proposed integrating the search state method and the extreme individuals method for JADE. The effectiveness of the proposed method was shown by optimizing 13 benchmark functions. In the future, we plan to investigate the control for the noisy function (f_7) where the performance of the proposed method was not good.

References

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